Subbotin Graphical Models for Extreme Value Dependencies with Applications to Functional Neuronal Connectivity

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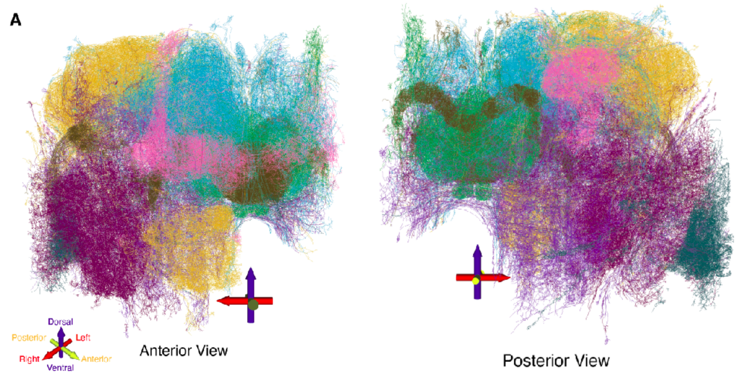
Houston Neuronex

Fruitfly Hemibrain Modularity

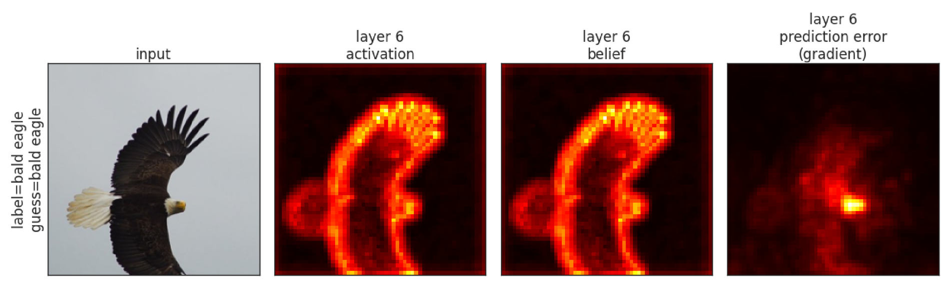
- **Dr. Xaq Pitkow & Dr. Alex Kunin, Baylor College of Medicine; Dr. Krešimir Josić, University of Houston**.
- Overview of modularity and anatomy in fruitfly Hemibrain dataset; clustering using a multi-resolution algorithm.
  - At coarsest scale, modularity is maximized by 8 cluster partition.
  - Study learned community structure vs. identified anatomical partitions.



Morphological renderings of sample of 120 neurons from each cluster (88 from cluster 8), color-coded by identity.

Predictive Coding & Backpropogation

- **Dr. Robert Rosenbaum, University of Notre Dame**.
- Relationship between predictive coding & backpropagation for training feedforward artificial neural networks on supervised learning tasks.

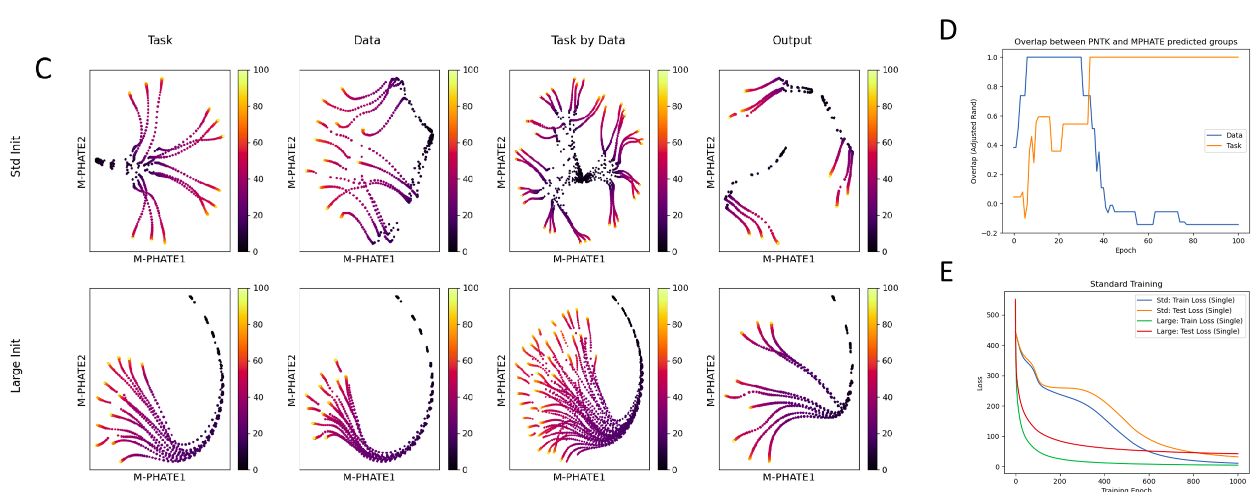


Magnitude of activations, beliefs, and prediction errors in CNN pre-trained on ImageNet.

- Prediction errors approximated by gradients of the loss.
- Large prediction errors can occur, even when image & network guess consistent with label.
  - Contradicts interpretation of prediction errors as measurements of "surprise."
  - e.g., eagle's head has high error, even though white head determines its classification.

Dynamic Routing Tasks for Neural Networks

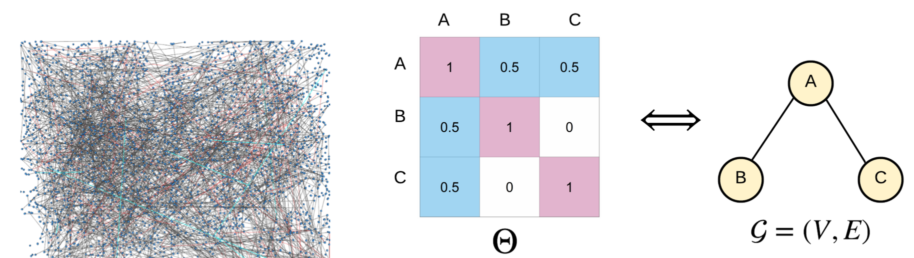
- **Dr. Ryan Pyle & Dr. Ankit Patel, Baylor College of Medicine**.
- Dynamic routing task: task inputs determine the routing pattern, data inputs determine what is to be routed.
  - **C**: MPHATE showing how hidden network states evolve.
    - High initialization learns each routing independently, while standard initialization learns a complex representation that re-uses relevant information across related tasks or stimuli.
  - **D**: Agreement between the PNTK predictions and the MPHATE clusterings.
  - **E**: Independent representations (high init) learn tasks quicker; shared representations (standard init) take longer, but eventually generalize better.



Subbotin Graphical Models

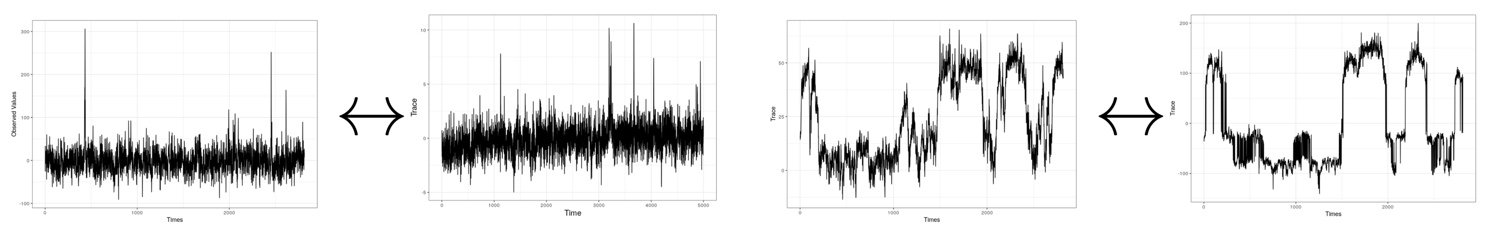
Background

- Estimating intrinsic functional connectivity using neuronal activity data from two photon calcium imaging.
  - Pairwise statistical relationships in contemporaneous neuronal activity.
- **Probabilistic graphical models**: sparse, data driven conditional dependency structures in high-dimensional data.



$$\Theta_{bc} = 0 \iff B \perp C \mid A$$

- **Challenge**: neuron firing in calcium imaging data represented by spikes, i.e. scarce extreme values.
  - Typical exponential family graphical models typically fit to mean, not extremes.
  - Multi-step pre-processing leads to data loss, error propogation.
- Develop new class of graphical models for single-step analysis data where the important information lies in rare extreme outliers.



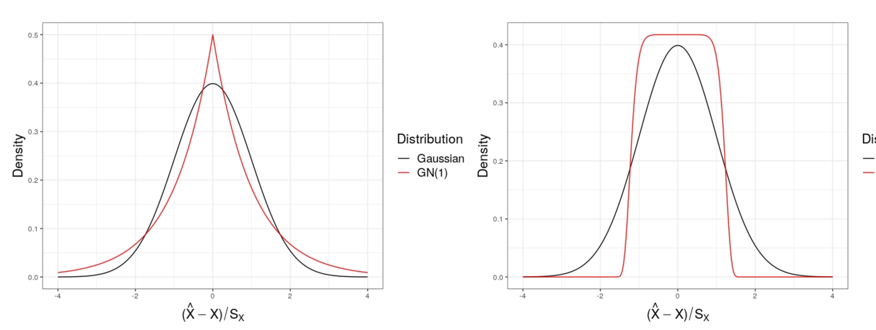
Expected vs. typical estimated functional connections from graphical models.

Formulation

- Utilize ideas from **robust modeling**.
  - Parameter estimates less sensitive to rare extreme outliers.
- Based around **Subbotin distribution**:

$$f(\epsilon, \nu) = \frac{\nu}{2\sigma\Gamma(1/\nu)} e^{-\left|\frac{\epsilon}{\sigma}\right|^{\nu}}$$

  - $\nu = 2$: Gaussian distribution.
  - $\nu < 2$: heavy-tailed, larger tolerance of predicting extreme as non-extreme.
- **Extreme value problem**: use Subbotin distribution with $\nu > 2$.
  - Thin-tailed, forces model to more accurately predict extreme values.



Left: Subbotin distribution, $\nu = 1$. **Right:** Subbotin distribution, $\nu = 10$.

- Construct joint graph distribution from node conditionals:

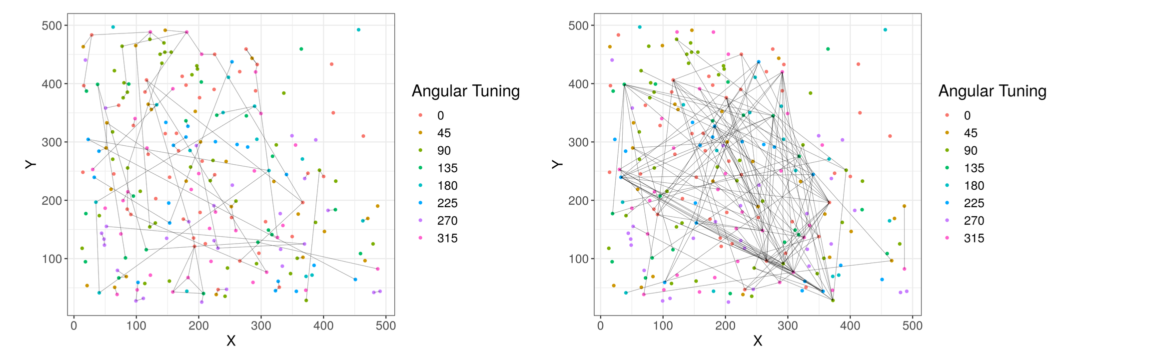
$$f(x_i|\mathbf{x}_{-i}) = \frac{\nu}{2\Gamma\left(\frac{1}{\nu}\right)} e^{-(x_i - (\Sigma_{j\neq i}\boldsymbol{\theta}_{ij}x_j))^{\nu}} \rightarrow$$

$$f(\mathbf{x}) = \exp\left(\Sigma_{i=1}^{p}\left(-(x_i - \Sigma_{j<i}\theta_{ij}x_j)^{\nu} + (\Sigma_{j<i}\boldsymbol{\theta}_{ij}x_j)^{\nu}\right) - A(\boldsymbol{\Theta})\right)$$

- Neighborhood selection approach for estimation: ℓ_ν norm regression.

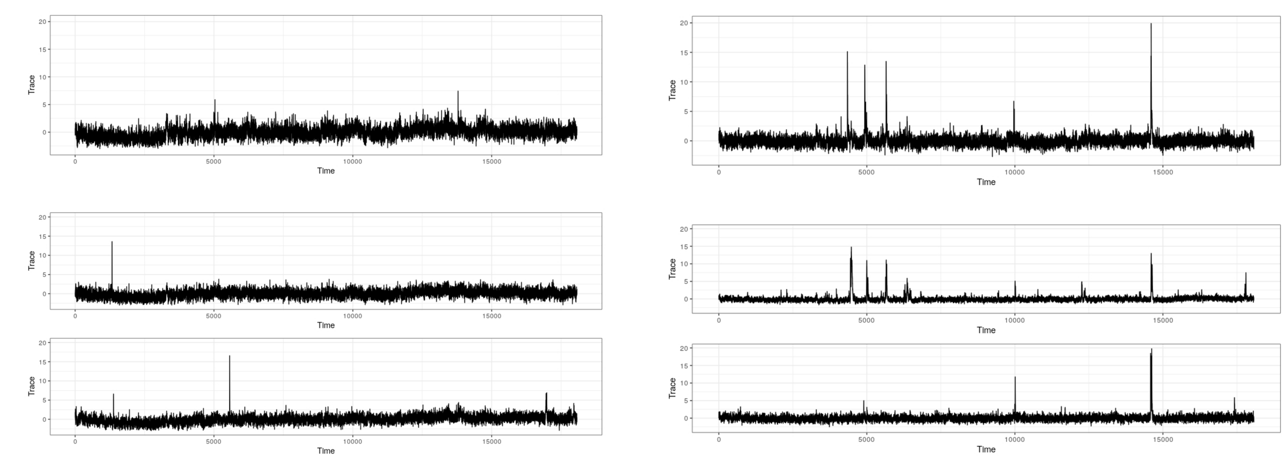
Empirical Studies

- Calcium imaging scan from Allen Brain Atlas Brain Observatory.
- 227 neurons in visual V1 cortex during drifting grating stimulus.
- Compare estimated functional connectivity networks from Subbotin Gaussian graphical models.



Functional connectivity graphs from Gaussian (top) and Subbotin (bottom) graphica

- Visualize fluorescence traces of hub neurons, edge neighbors.
- Subbotin edges contain contemporaneous spikes; Gaussian edges do



Hub neuron traces (top) and edge neighbor traces (bottom) from Gaussian (left) and Subbotin (right) graphical model functional connectivity estimates.

- Compare proportion of estimated functional connections from graph functional connectivity models with same neuron tuning category.
- Subbotin graphical model does best at matching tuning categories.

Model	Angular Tuning	Frequency Tuning
Subbotin ($\nu = 10$)	**0.578**	**0.642**
Gaussian Glasso	0.429	0.449
Quantile ($q = 0.99$)	0.494	0.491
Copula ($b = 30$)	0.292	0.262
Hawkes	0.140	0.335
Transfer Entropy	0.243	0.193
VAR	0.112	0.387
Linear-Nonlinear	0.460	0.434

Conclusion

- Develop & study new class of graphical model distribution for cond dependencies with respect to extreme value observations.
- More sensible functional connectivity estimates than current graphi modeling techniques for two-photon calcium imaging data.
- **Future work**: intrinsic functional connectivity network architectu larger data, relationship between functional and structural connecti

Acknowledgements

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